Scientific report on project implementation

“Approximation by nonlinear operators of max-product type and distance methods in fuzzy number theory, applied to signal and image processing”
(PN-II-ID-PCE-2011-3-0861)

during the period January-December 2014

Research team consists of Adrian Ion Ban, full professor, Alexandru Mihai Bica, full professor, Lucian Coroianu, assistant professor and Sorin Gal full professor, all from the Department of Mathematics and Computer Science, University of Oradea.

Some of the papers produced in this stage were written by international collaboration with Barnabas Bede, from DigiPen Institute of Technology, Department of Mathematics, Redmond, WA, USA, Marek Gagolewski and Przemyslaw Grzegorzewska from Systems Research Institute, Polish Academy of Sciences and the Faculty of Mathematics and Computer Science of the Technical University of Warsaw, Luciano Stefanini from the University "Carlo Bo" of Urbino, Aireza Khastan from the Institute for Advanced Studies in Fundamental Sciences in Zanjan, Iran, M. Adabitabar Firozja from Azad University (English name is Qaemshahr Branch, Islamic Azad University) in Iran and Tahereh Houlari Damghan University in Iran.

At this stage, the team’s members focused on topics related to objectives 1, 2, 4, 8, 9, 10, 13, 14, 16 and 17 of the project. Have been published or accepted for publication a total number of 10 papers, from which 7 ISI papers, one in a journal reviewed in Zentralblatt fur Mathematik and Mathematical Reviews (well-known in approximation theory), 1 in ISI Proceedings and 1 in Proceedings published by Springer-Verlag publishing house, 2014. Also during the year 2014, were developed and submitted for publication a number of 7 papers, which are expecting for referees’ decisions. In addition, we have continued to work on the drafts of the two research monographs intended to be published at Imperial College Press, World Scientific Publisher and Springer, and are in various stages of development other 4 papers.

Concerning participations to international and national conferences and research seminars in 2014, we mention the following activity of Lucian Coroianu:

a) During 13.05.2014-02.08.2014, he was visiting professor at the "Research Systems Institute of the Polish Academy of Sciences" in Warsaw, with which occasion presenting two lectures on recent results obtained in the project, in approximation of fuzzy numbers, entitled
"Recent results in the ranking of fuzzy numbers" (on 05.26.2014, at the said institution) and "Standard and interactive arithmetic on fuzzy numbers" (in 06/10/2014, Faculty of Mathematics and Computer Science at the Technical University of Warsaw).

b) between 15 and 19 July, 2014, he attended the "15-th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, Montpellier, France", during which he presented a paper on ordering fuzzy numbers, written in collaboration with Adrian I. Ban (paper subsequently published in the conference proceeding, at Springer-Verlag publisher, in 2014, see the paper B2).

c) between 18 and 21 September, 2014, he attended with a presentation the ROMAI conference CAIM, 2014, held in Bacau, during which he received the "Adelina Georgescu" prize in applied mathematics for his work on the study of fuzzy numbers.

Papers corresponding to the activity on 2014

A. ISI papers published or accepted for publication in 2014


   In this paper we consider the approximation of fuzzy numbers with max-product operators of Bernstein type, preserving their different characteristics such as ambiguity, expectancy value interval expectancy, width, core, etc. The, we deduce the order of approximation \( 1 / n \) in the norm of space \( L^1 \) space. One answer to the objectives 8, 10, 13 and 14 in the project.

2. Coroianu, L. and Gal, SG, Saturation and inverse results for the max-product Bernstein operator, Periodica Mathematica Hungarica, 69 (2014), pp. 126-133, Impact factor 0.379, Relative Score of Influence 0.577;

   We show that the order of saturation for the max-product Bernstein operator is \( 1 / n \), such that th constant functions are the only ones essential order of approximation smaller than \( 1 / n \). Then, it is shown that on strict subintervals, the functions with approximation order \( 1 / n \) are local Lipschitz functions on those subintervals. One answers to Objective 1 of the project.


   By analogy with the probabilistic approach of the classical Bernstein polynomials, in this paper we first give a proof of the uniform convergence for max-product type Bernstein
operator, by using the possibilistic theory. This approach that interprets max-product Bernstein operators as the expectancy value of a possibilistic Bernoulli distribution, not only offers a good explanation of this operator max-product, but also allows to extend the method to other operators of max-product Bernstein-types. We answer to Objectives 1, 2, 3 and 5 of the project.


In the class of strictly positive functions, the following strong localization type result for the max-product operator Bernstein is proved: if f and g coincide on a strict subinterval, then for sufficiently large n, the max-product Bernstein operator of degree n, attached to f on a subinterval no matter how close to the original, coincides with the max-product Bernstein operator attached to the function g. This result allows local approximation of functions bounded strictly positive with very high accuracy, having potential applications in image processing and fuzzy approximation numbers. This answers to Objective 1 of the project.


We characterize the set of real parameters associated to a fuzzy number, represented in a general form, which includes the most important features, with the property that there is at least a trapezoidal fuzzy number that keeps the parameter considered. Also, we study the problem of uniqueness and the continuity properties. The 9th and 12th goals of the project are thus achieved.


Fuzzy arithmetic based on Zadeh's extension principle, has the disadvantage that certain properties that sometimes works for normal operation cannot be extended to the case of fuzzy numbers. Therefore, many authors have tried to replace the principle of expansion by using triangular norms, which is an even more general approach. Recently, this method was extended further using the so-called possibility distributions. In this paper, in the case of addition, necessary and sufficient conditions are found such that the standard addition and the addition of two fuzzy numbers based on possibility distributions coincide. Thus, there is an answer to Fuller, Carlsson and Majlender' open question. This article touches a part of Objective 16 of the project.


We show that in most cases, an ordering index can be reduced to an index of ordering of
very simple form and that generates an equivalent order on the set of fuzzy numbers. On the set of fuzzy trapezoidal numbers, these indexes can be determined effectively and, moreover, can be extended to order arbitrary fuzzy numbers. The article is a very significant contribution to Objective 11 of the project.

B. Papers published or accepted in Proceedings or in reviewed journals in Math. Reviews and Zentr. fur Mathematik in 2014

1. Coroianu, L. and Gal, S.G, Localization results for the non-truncated max-product sampling and Operators based on Sinc-type Fejer kernels, Demonstratio Mathematica, Walter de Gruyter der Publ., Accepted for publication in issue 3 or 4 of volume 48 (2015), SCImago Journal Rank (SJR): 0.189

For non-truncated max-product operators based on sinc type and Fejer type kernels, in the strictly positive function class, we prove the following strong localization result: if f and g coincide on a strict subinterval, then for n large enough, the max-product operator of degree n of f, attached to a subinterval no matter how close to the original, coincides with the max-product operator attached to g. This result allows local approximation of bounded, strictly positive functions, with very high accuracy, having potential applications in signal theory. Responds to the objectives 1, 4 and 17 of the project.


Excepting equivalent orderings, we can characterize the ordering indexes defined on triangular fuzzy numbers that verify the basic requirements (Wang and Kerr's) in ordering fuzzy numbers. Article contribute to achieving target 11 of the project.


In this paper we determine approximation algorithms which calculate the approximation of the fuzzy numbers by fuzzy numbers with 2 segmental linear segments which preserves both the support and the core. Applications are obtained by using fuzzy numbers generated by Euler-beta distributions. This paper contributes to the objective 8 of the project.

C. Papers submitted for publication in 2014

In this paper, we introduce extended L-R fuzzy numbers. Then, with respect to a weighted metric in $L^2$ and considering the functions L and R, one generalizes important parameters such as ambiguity, value, core or interval of expectancy. It is shown that with respect to any of these parameters, there exists a unique approximation by L-R type fuzzy numbers that preserve the parameter. Algorithms to compute these approximations are proposed. This paper generalizes both the parametric and trapezoidal approximation. This paper contributes to the objective 8 of the project.


We determine the class of the real parameters written in a general form, that in particular give us important features with the property that there is at least a symmetrical triangular fuzzy number preserving this parameter. It then demonstrates approximation which The uniqueness of the approximation and the calculation method is determined. Finally, the properties of invariance with respect to scalars and translations, the additivity and continuity of the approximation operators are obtained. This paper contributes to the objectives 8, 9 and 12.


We prove that the fuzzy transform (F-transform, for short) preserve the support of the fuzzy number, the core of a fuzzy number and the quasi-concavity. The general approximation normalizing fuzzy numbers which besides linear uniform convergence, in addition preserve and important features of fuzzy numbers. The results are closely to the 8.9, 13 and 14 objectives of the project.


One generalizes the segmental linear fuzzy numbers. Algorithms are proposed for approximation (which exists, unique) with respect to the Euclidean metric. For the approximation operator is proved that is invariant with respect to scalars and translations and that are non-expansive. Then, when $n$ tends to infinity one prove the convergence of the sequence of approximations with respect to the Euclidean metric and the convergence with respect the important characteristics. This paper contributes to the objectives 8 and 9, 14 and 14 of the project.


Most proposed fuzzy approximation operators proposed verify the property that are piecewise additive and positively homogeneous, in the sense that the space of fuzzy numbers is covered by a family of convex cones, such that one each cone, the approximation operator is additive and positively homogeneous. Note that the approximate solution can be obtained.
as a quadratic optimization problem (quadratic program). In this paper, we study the quadratic optimization problem to find further results on the behavior of fuzzy approximation operators. It answers to the objective 9 of the project.


One determine the class of real parameters in a general form that in particular gives us important features with the property that there is at least a triangular fuzzy number conserving this parameter. For the obtained approximation operator is studying the properties of invariant scalar and translations and continuity. This paper contributes to the objectives 8, 9 and 12 of the project.


Starting from the classical Weierstrass functions, non-differentiable everywhere, in this paper we introduce the max-product Weierstrass functions, for which it is shown that the set points of non-differentiability is infinite, countable, nowhere dense and of Lebesgue measure zero. It is also studied some fractal properties of these functions. The paper answers to Objective 1 of the project.

D. Research monographs in preparation in 2014

1. Lucian Coroianu and Sorin Gal (in collaboration with Barnabas Bede) wrote about 95% (i.e. 334 pages) from the manuscript of the research monograph "Approximation by Max-Product Type Operators and Applications", requested by the series "Approximation and Decomposition" publisher "Imperial College Press, World Scientific Publisher". It is expected to be completed and submitted during 2015.

2. Adrian Ioan Ban and Lucian Coroianu (in collaboration with Przemyslaw Grzegorzewska) wrote about 75% from the research monograph "Approximations of Fuzzy Numbers and Their Applications", prepared to be published at Springer, in the series "Studies in fuzziness and Soft Computing ". At this time the manuscript is 360 pages. It is expected to be completed and submitted in 2015.

E. Papers in preparation in 2014

1. Bede, B., Coroianu, L. and Gal, SG, Approximation and shape preserving properties of the tensor product Bernstein max-operator of two variables in the square with applications to image processing.


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